$\frac{14}{25}$

Birzeit University-Mathematics Department
Math132-Calculus II

Second Semester 2013/2014

Second Exam
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Section: Sat: 11:00-11:50.

Question 1. (19 points) Circle the correct answer:

(1) The sequence $a_n = e^{-n^{1/n}}$

(a) Converges to e.

(b) Converges to e^{-1}.

(c) Converges to 1.

(d) Diverges.

$(e^n)^{1/n}$ $\quad \frac{1}{e^{n \cdot 1/n}}$ $\quad \frac{1}{e}$ $\quad e^{-1}$ $\quad \frac{1}{e}$

(2) $\int_2^\infty \frac{2dx}{x^2-1}$

(a) Converges to 1.

(b) Converges to $\ln 3$.

(c) Converges to 0

(d) Diverges.

$\frac{2}{x^2-1}$ $\quad \int_2^\infty \frac{2\,dx}{x^2-1}$

(3) The series $\sum_{n=0}^{\infty} \frac{1}{e^n+e^{-n}}$

(a) Converges by integral test.

(b) Diverges by integral test.

(c) Diverges by nth term test.

(d) None of the above.

$\frac{1}{1+1} = \frac{1}{2} + \cdots \quad (\cosh n)^{-1}$ $\quad \int \frac{1}{e^n + e^{-\infty}}$ $\quad \frac{1}{\infty + 0}$

(4) The series $\sum_{n=1}^{\infty} \frac{n}{e^n}$

(a) Converges by nth term test.

(b) Diverges by nth term test.

(c) Diverges by ratio test.

(d) Converges by nth-root test.

$\frac{n}{e^n}$ $\quad \sqrt[n]{\frac{n}{e^n}}$ $\quad \frac{n^{\frac{1}{n}}}{e}$ $\quad \frac{1}{e}$

$\frac{n+1}{e^{n}}$ $\quad \frac{n}{en} + \frac{1}{en}$ $\quad \frac{1}{e} + 0 < 1$

$n^{\frac{1}{n}} = 1$

$\sqrt[n]{n} = 1$

(5) The series $\sum_{n=0}^{\infty} \frac{3^n}{2^n+3^n}$

(a) Converges by integral test.
(b) Diverges by direct comparison with $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$.
(c) Converges by nth term test.
(d) Diverges by nth term test.

(6) The interval of convergence of the series $\sum_{n=2}^{\infty} \frac{(x-1)^n}{n \ln n}$ is

(a) $[0, 2]$.
(b) $[0, 2)$.
(c) $(0, 2)$.
(d) $(0, 2]$.

(7) One of the following series converges absolutely

(a) $\sum_{n=1}^{n} \frac{(-1)^n}{n}$.
(b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n^2-1}}$.
(c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$.
(d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$.

(8) $\sum_{n=0}^{\infty}(e^{-n} - e^{-(n+2)}) =$

(a) $1 + e^{-1}$.
(b) $e^{-1} + e^{-2}$.
(c) e^{-1}.
(d) None of the above.

(9) The Maclaurin series generated by $f(x) = 3^x$ is

(a) $\sum_{n=0}^{\infty} \frac{3^n}{n!}$.
(b) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$.
(c) $\sum_{n=0}^{\infty} \frac{(\ln 3)x^n}{n!}$.
(d) $\sum_{n=0}^{\infty} \frac{(\ln 3)^n x^n}{n!}$.

$\int \frac{\ln n}{\sqrt{n}}$ $\frac{1}{n^2}$ converge

(10) $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}} =$

(a) Converges by integral test.

(b) Converges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{n^2}$.

(c) Diverges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{n}$.

(d) Diverges by nth term test.

$\frac{\ln n}{\sqrt{n}}$ $\frac{\ln n}{\sqrt{n}} \cdot \frac{1}{\ln}$

(11) $\int_2^{\infty} \frac{2+\sin x}{x-1} dx$

(a) Converges to 0.

(b) Diverges by limit comparison with $\int_2^{\infty} \frac{dx}{x-1}$.

(c) Diverges by direct comparison with $\int_2^{\infty} \frac{dx}{x}$.

(d) None of the above.

$\ln(x-1)$

$\ln x$

$\int \frac{5}{5} = \int 2+\sin x$

$= (2-\cos x) \Big|_0^b$

(12) $\frac{\pi}{2} - \frac{\pi^3}{2^3(3!)} + \frac{\pi^5}{2^5(5!)} - \cdots + (-1)^n \frac{\pi^{2n+1}}{2^{2n+1}(2n+1)!} + \cdots =$

(a) 0.

(b) 1.

(c) -1.

(d) ∞.

(13) The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{n^3(x-2)^n}{2^n}$ is

(a) 0.

(b) 1.

(c) $\frac{1}{2}$.

(d) 2.

$n\left(e^{\left(\frac{-1}{n}\right)} - 1\right)$

(14) The sequence $a_n = n(e^{-1/n} - 1)$

(a) Diverges.

(b) Converges to -1.

(c) Converges to 1.

(d) Converges to e^{-1}.

$n(e^{-1/n} - 1)$

$n\left(\frac{1}{e^{1/n}} - 1\right)$

$\frac{n}{e^{1/n}} - n$

$\frac{1}{\quad} - 1$

$$\frac{(n+1)!\,e^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{n!\,e^{n}}$$

$$\frac{e(n+1)}{2(n+1)} = \frac{e}{2} < 1$$

(15) $\sum_{n=0}^{\infty} \frac{n! e^n}{(2n)!}$

(a) Converges by ratio test.
(b) Converges by nth term test.
(c) Diverges by ratio test.
(d) Ratio test fails.

(16) The error in the approximation $\frac{1}{1+x^2} \approx 1 - x^2 + x^4 - x^6 - x^8$ in the interval $[-0.1, 0.1]$ is less than

(a) 1×10^{-10}.
(b) 1×10^{-9}.
(c) 1×10^{-8}.
(d) 1×10^{-7}.

(17) The Maclaurin series generated by $f(x) = \frac{x^2}{1+x}$ is

(a) $\sum_{n=0}^{\infty} (-1)^n x^n$.
(b) $\sum_{n=0}^{\infty} (-1)^n x^{n+1}$.
(c) $\sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!}$.
(d) $\sum_{n=0}^{\infty} (-1)^n x^{n+2}$.

(18) Suppose that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = L$ then

(a) $\frac{3}{4} < L < 1$.
(b) $1 < L < \frac{5}{4}$.
(c) $\frac{1}{4} < L < \frac{3}{4}$.
(d) None of the above.

(19) The series $\sum_{n=1}^{\infty} \frac{\tanh n}{n^2}$

(a) Diverges.
(b) Converges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.
(c) Diverges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.
(d) Is an alternating series.

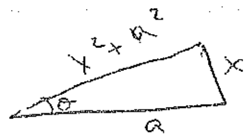
Question 2(6 points) Solve the integrals

(a) $\int \frac{x^2}{(x^2+1)^{3/2}}dx$ using trigonometric substitution.

$$\int \frac{x^2}{\sqrt{(x^2+1)^3}} =$$

$$\frac{x^2}{(x^2)^{3/2}+1} = \int \frac{x^2}{x^3+1}$$

$$= \int \frac{\cancel{x}}{x^3+1}$$



$x = a\tan\theta$

$dx = a\sec^2\theta$

$\sqrt{x^2+a^2} =$

(b) $\int \frac{x+1}{x(x^2+1)}dx$

$$\int \frac{x}{x^2+1} - \int \frac{1}{x^2+1}$$

$$\int \frac{x+1}{x(x^2+1)}\,dx = \frac{(x^2+1)A}{(x^2+1)x} + \frac{Bx+C}{x^2+1}\frac{(x)}{(x)} = \int \frac{1}{x} - \int \frac{x+1}{x^2+1} = \boxed{\ln x - \tfrac{1}{2}\ln(x^2+1) - \tan^{-1}x}$$

$2x(A+B)+0+c$

$$= \frac{A(x^2+1)+Bx^2+cx}{x(x^2+1)}$$

$$\frac{Ax^2+A+Bx^2+cx}{x(x^2+1)} = \frac{x^2(A+B)+A+cx}{x(x^2+1)}$$

$1 = A+B+A+c = 2A+B$

$1 = 2(A+B) + \cancel{A} + c = 2A+2B+\cancel{A}+c = 2A+2B+c$

$0 = 2A+2B.$

$-2A = 2B$

$\boxed{-A=B}$

$1 = 2A - A = A \longrightarrow \boxed{1 = A}$

$\boxed{B=-1}$

$1 = 2(1)+2(-1)+c$

$1 = 2+-2+c$

$\boxed{c=1}$

$$\Rightarrow \ln x - \frac{1}{2}\ln(x^2+1) - \tan^{-1}x$$